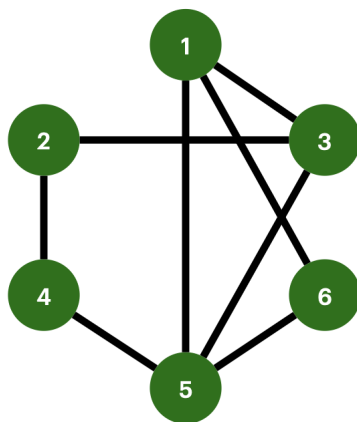# Grade 9/10 Math Circles
October 25, 2023
Graph Theory - Problem Set

Graph Basics

Graph A

1. Find the vertex and edge set of Graph A.

2. For Graph A:

   (a) Find the neighbours of vertex 1 and vertex 5

   (b) Find the degree of vertex 1 and vertex 5

   (c) What do you notice about the degree and the neighbours of a given vertex? Why is this the case?

3. For Graph A:

   (a) Find a walk from vertex 1 to 4

   (b) Find a path from vertex 2 to 5

   (c) Find a cycle

   (d) Find a spanning tree

4. Describe a graph (with vertex and edge sets) that, when drawn, can be in the shape of something fun!

Word Graphs

1. Create a word graph using the following words:
   BARN, BEND, BENT, BERN, FERN, LAND, LEND, LENT, RENT

2. Find a potential path in a word graph from MATH to TEAM.
   Hint: Try passing through the word PEAS along the way!

3. What would the word graph of {a, b, c, d, e, ..., v, w, x, y, z}) look like?

4. A star is a graph each vertex (aside from the 'centre') has degree 1 and is connected by an edge to the 'centre'. An example is given (Graph B).

   Graph B

   (a) Call a star with k non-'centre' vertices S_k. Draw the S_5 graph.

   (b) Create a word graph that has the shape of S_4.

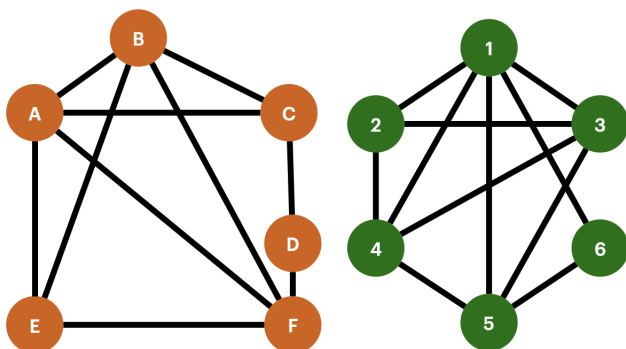   (c) **Challenge:** What properties must the words satisfy in order to create an S_k word graph?
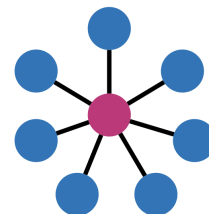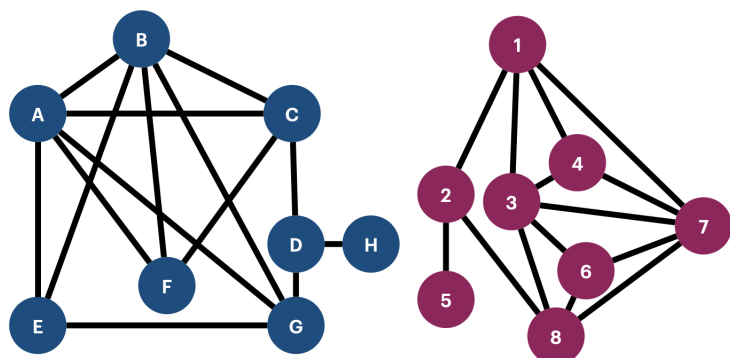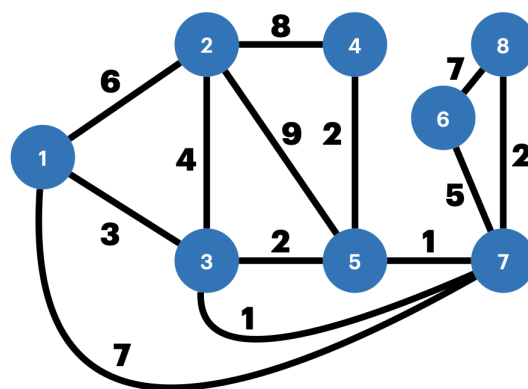
Isomorphic Graphs

Pair A

1. Prove the graphs in Pair A are isomorphic by providing an isomorphism between them.

2. **Challenge:** Prove that the graphs in Pair B are not isomorphic.

3. Draw all non-isomorphic graphs which have 6 vertices and less than 4 edges.

   Hint: There should be 9 such graphs.

Pair B

Handshaking Lemma

1. Confirm that your graph from Graph Basics Q4 satisfies the Handshaking Lemma.

2. A graph is k-regular if each vertex has degree k. Find the number of edges in a 3-regular graph with 10 vertices.

3. Find the number of vertices in a 4-regular graph with 72 edges.

Prim's Algorithm

1. Find an MST of Graph C using Prim's Algorithm.

Graph C